THE REZNICHENKO PROPERTY AND THE PYTKEEV PROPERTY IN HYPERSPACES

LJUBIŠA D.R. KOČINAC

Abstract. We investigate two closure-type properties, the Reznichenko property and the Pytkeev property, in hyperspace topologies.

1. Introduction

Let X be a space (we suppose that all spaces are Hausdorff). For a subset A of X and a family $\mathcal{A}$ of subsets of X we put $A^c = X \setminus A$ and $\mathcal{A}^c = \{A^c : A \in \mathcal{A}\}$. By 2^X we denote the family of all closed subsets of X. If A is a subset of X, then we write

$$A^- = \{F \in 2^X : F \cap A \neq \emptyset\},$$
$$A^+ = \{F \in 2^X : F \subset A\}.$$

There are many known topologies on 2^X. The most popular among them is the Vietoris topology $\mathsf{V} = \mathsf{V}^- \vee \mathsf{V}^+$, where the *lower Vietoris topology* V^- is generated by all sets A^-, $A \subset X$ open, and the *upper Vietoris topology* V^+ is generated by sets B^+, B open in X.

Let Δ be a subset of 2^X. Then the *upper Δ-topology*, denoted by Δ^+ (and first studied in abstract in [19] and then in [5]), is the topology whose subbase is the collection

$$\{(D^c)^+ : D \in \Delta\} \cup \{2^X\}.$$

We consider only such subsets Δ of 2^X which are **closed for finite unions** and **contain all singletons**. In that case the above collection is a base for Δ^+ because we have

$$(D_1^c)^+ \cap (D_2^c)^+ = (D_1^c \cap D_2^c)^+ = ((D_1 \cup D_2)^c)^+ \text{ and } D_1 \cup D_2 \in \Delta.$$

Two important special cases, in which we are especially interested in this paper, are $\Delta = \mathbb{F}(X)$ – the family of all finite subsets of X, and $\Delta = \mathbb{K}(X)$ – the collection of compact subsets of X. The $\mathbb{F}(X)^+$-topology will be denoted by Z^+ and the $\mathbb{K}(X)^+$-topology by F^+. The F^+-topology is known as the

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upper Fell topology (or the *co-compact topology*) [10]. The *Fell topology* F on 2^X is the topology $\mathsf{V}^- \vee \mathsf{F}^+$.

We investigate two closure type properties of 2^X, the Reznichenko property and the Pytkeev property, which are intermediate between sequentiality and the countable tightness property that have been studied in [4] and [12].

Let us fix some terminology and notation that we need.

Let $\mathcal{A}$ and $\mathcal{B}$ be sets whose members are families of subsets of an infinite set X. Then (see [24], [14]):

$\mathsf{S}_1(\mathcal{A}, \mathcal{B})$ denotes the selection principle:

> For each sequence $(A_n : n \in \mathbb{N})$ of elements of $\mathcal{A}$ there is a sequence $(b_n : n \in \mathbb{N})$ such that for each n $b_n \in A_n$, and $\{b_n : n \in \mathbb{N}\}$ is an element of $\mathcal{B}$.

$\mathsf{S}_{fin}(\mathcal{A}, \mathcal{B})$ denotes the selection hypothesis:

> For each sequence $(A_n : n \in \mathbb{N})$ of elements of $\mathcal{A}$ there is a sequence $(B_n : n \in \mathbb{N})$ of finite sets such that for each n $B_n \subset A_n$, and $\bigcup_{n\in\mathbb{N}} B_n$ is an element of $\mathcal{B}$.

If $\mathcal{C}$ is a family of subsets of a space X then an open cover $\mathcal{U}$ of X is called a $\mathcal{C}$-*cover* if each $C \in \mathcal{C}$ is contained in an element of $\mathcal{U}$. $\mathbb{F}(X)$-covers and $\mathbb{K}(X)$-covers are customarily called ω-*covers* and k-*covers*, respectively. We suppose that $\mathcal{C}$-covers we consider are non-trivial, i.e. that X does not belong to the cover.

For a topological space X and a point $x \in X$ we denote:

1. $O\mathcal{C}$ – the family of (open) $\mathcal{C}$-covers of X.
2. Ω – the family of ω-covers of X.
3. $\mathcal{K}$ – the family of k-covers of X.
4. $\Omega_x = \{A \subset X : x \in \overline{A} \setminus A\}$.

We also need the notion of groupability (see [17]).

A countable $\mathcal{C}$-cover $\mathcal{U}$ of a space X is *groupable* if there is a partition $(\mathcal{U}_n : n \in \mathbb{N})$ of $\mathcal{U}$ into pairwise disjoint finite sets such that for each $C \in \mathcal{C}$, for all but finitely many n there is a $U \in \mathcal{U}_n$ such that $C \subset U$. A countable set $A \in \Omega_x$ is *groupable* if there is a partition $(B_n : n \in \mathbb{N})$ of A into finite sets such that each neighborhood of x has nonempty intersection with all but finitely many B_n.

Let us denote:

5. $O\mathcal{C}^{gp}$ – the family of groupable $\mathcal{C}$-covers of X.
6. Ω^{gp} – the family of groupable ω-covers of X.
7. $\mathcal{K}^{gp}$ – the family of groupable k-covers of X.
8. Ω_x^{gp} – the family of groupable elements of Ω_x.

Let us mention that considering the space 2^X, $\Delta \subset 2^X$ and $S \in 2^X$ we shall use the symbol Δ_S^+ to denote Ω_S with respect to Δ^+-topology on 2^X. We use Ω_S and $\mathcal{K}_S$ studying the Z^+- and F^+-topology on 2^X. Similarly for $(\Delta_S^+)^{gp}$ Ω_S^{gp} and $\mathcal{K}_S^{gp}$.

2. The Reznichenko property

In 1996, Reznichenko introduced (in a seminar at Moscow State University) the following property for a space X: For each $A \subset X$ and each $x \in \overline{A} \setminus A$ there is a countable infinite family $\mathcal{A}$ of finite pairwise disjoint subsets of A such that each neighborhood of x meets all but finitely many elements of $\mathcal{A}$. It is the same as to say that each countable element of Ω_x is a member of Ω_x^{gp}. This property was studied further in [18] (under the name *weakly Fréchet-Urysohn*), [9]. In [16] and [17] this property was called the *Reznichenko property* and function spaces $\mathsf{C}_p(X)$ having this property were studied (see also [23]). In [15] this property was considered in function spaces $\mathsf{C}_k(X)$.

Evidently, if a space X has the Reznichenko property then it has countable tightness.

Here we investigate the Reznichenko property in hyperspaces.

Theorem 1. *For a space X and a family $\Delta \subset 2^X$ (closed for finite unions and containing all singletons) the following statements are equivalent:*

(1) *$(2^X, \Delta^+)$ has the Reznichenko property;*
(2) *For each open set $Y \subset X$ and each open Δ-cover $\mathcal{U}$ of Y there is a sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of finite pairwise disjoint subsets of $\mathcal{U}$ such that each $D \in \Delta$ belongs to some $U \in \mathcal{U}_n$ for all but finitely many n.*

Proof. (1) $\Rightarrow$ (2): Let Y be an open subset of X and let $\mathcal{U}$ be an open Δ-cover of Y. Then $\mathcal{A} := \mathcal{U}^c$ is a subset of 2^X and $Y^c \in Cl_{\Delta^+}(\mathcal{A})$. Indeed, let $D \in \Delta$ be a subset of Y. There is a $U \in \mathcal{U}$ such that $D \subset U \subset Y$ and thus $Y^c \subset U^c \subset D^c$, i.e. $U^c \in (D^c)^+$ and $Y^c \in (D^c)^+$. So, $Y^c \in (D^c)^+ \cap \mathcal{A}$, that is $Y^c \in Cl_{\Delta^+}(\mathcal{A})$. Apply (1) to find a sequence $(\mathcal{A}_n : n \in \mathbb{N})$ of finite pairwise disjoint subsets of $\mathcal{A}$ such that each Δ^+-neighborhood of Y^c intersects $\mathcal{A}_n$ for all but finitely many n. For each n let $\mathcal{U}_n = \mathcal{A}_n^c$. The sets $\mathcal{U}_n \subset \mathcal{U}$ are pairwise disjoint (because $\mathcal{A}_n$'s are) and witness for $\mathcal{U}$ that Y satisfies (2). Let $D \subset Y$ be an element from Δ. Then $(D^c)^+$ is a Δ^+-neighborhood of Y^c, so that there is n_0 such that $(D^c)^+ \cap \mathcal{A}_n \neq \emptyset$ for each $n > n_0$. So, for each $n > n_0$ there exists a set $A_n \in \mathcal{A}_n$ with $A_n \subset D^c$, i.e. $D \subset A_n^c \in \mathcal{U}_n$. This means that (2) holds.

(2) $\Rightarrow$ (1): Let $\mathcal{A}$ be a subset of 2^X and $S \in 2^X$ a point such that $S \in Cl_{\Delta^+}(\mathcal{A}) \setminus \mathcal{A}$. Then $\mathcal{U} := \mathcal{A}^c$ is a (non-trivial) Δ-cover of the open set $S^c \subset X$. Apply (2) to S^c and $\mathcal{U}$. One can choose a sequence $(\mathcal{V}_n : n \in \mathbb{N})$ of finite pairwise disjoint subsets of $\mathcal{U}$ such that for each $D \subset Y$ belonging to Δ for all but finitely many n there is a $V \in \mathcal{V}_n$ with $D \subset V$. Let for each n, $\mathcal{B}_n = \mathcal{V}_n^c$. Then the collection $\{\mathcal{B}_n : n \in \mathbb{N}\}$ witnesses (1). Let $(D^c)^+$ be a Δ^+-neighborhood of S. Then $S \subset D^c$ implies $D \subset S^c$ so that there is $m \in \mathbb{N}$ such that for all $n > m$ there exists a member $B_n \in \mathcal{B}_n$ with $D \subset B_n$, and consequently $B_n^c \in (D^c)^+$. This shows that $(D^c)^+ \cap \mathcal{B}_n \neq \emptyset$ for all but finitely many n and completes the proof of the theorem. $\square$

In fact, we shall prove a general result regarding the Reznichenko property in hyperspaces.

Theorem 2. *Let X be a space and let Δ and Σ be subsets of 2^X closed for finite unions and containing all singletons. Then the following statements are equivalent:*

(1) *2^X satisfies $\mathsf{S}_1(\Delta^+_A, (\Sigma^+_A)^{gp})$ for each $A \in 2^X$;*
(2) *Each open set $Y \subset X$ satisfies $\mathsf{S}_1(O\Delta, O\Sigma^{gp})$.*

Proof. (1) $\Rightarrow$ (2): Let $(\mathcal{U}_n : n \in \mathbb{N})$ be a sequence of Δ-covers of Y. Then $(\mathcal{U}_n^c : n \in \mathbb{N})$ is a sequence of subsets of 2^X and $Y^c \in Cl_{\Delta^+}(\mathcal{U}_n^c)$ for each $n \in \mathbb{N}$. Applying (1) we find a sequence $(U_n^c : n \in \mathbb{N})$ such that for each n $U_n \in \mathcal{U}_n$ and $\mathcal{F} = \{U_n^c : n \in \mathbb{N}\} \in (\Sigma^+_{Y^c})^{gp}$. There is a partition $\mathcal{F} = \bigcup_{n\in\mathbb{N}} \mathcal{F}_n$ of $\mathcal{F}$ such that each Σ^+-neighborhood of Y^c meets all but finitely many sets $\mathcal{F}_n$. For each n let $\mathcal{V}_n = \mathcal{F}_n^c$. The sets $\mathcal{V}_n$ are pairwise disjoint, finite subsets of $\{U_n : n \in \mathbb{N}\}$ and show that this set is a groupable Σ-cover of Y. Let $S \subset Y$ be a member of Σ. Then $(S^c)^+$ is a Σ^+-neighborhood of Y^c, so that there is n_0 such that $(S^c)^+ \cap \mathcal{F}_n \neq \emptyset$ for each $n > n_0$. So, for each $n > n_0$ there exists a set $F_n \in \mathcal{F}_n$ which is a subset of S^c, i.e. $S \subset F_n^c \in \mathcal{V}_n$. This means that $\{U_n : n \in \mathbb{N}\}$ is a groupable Σ-cover of Y, hence (2) holds.

(2) $\Rightarrow$ (1): Let $(\mathcal{A}_n : n \in \mathbb{N})$ be a sequence of subsets of 2^X such that a point $E \in 2^X$ belongs to $Cl_{\Delta^+}(\mathcal{A}_n) \setminus \mathcal{A}_n$ for each n. For each n put $\mathcal{U}_n = \mathcal{A}_n^c$. Apply (2) to the open set E^c and the sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of Δ-covers of E^c. We choose a sequence $(U_n : n \in \mathcal{U}_n)$ such that for each n $U_n \in \mathcal{U}_n$ and $\mathcal{G} = \{U_n : n \in \mathbb{N}\}$ is a groupable Σ-cover of E^c. Suppose that the partition $\mathcal{G} = \bigcup_{n\in\mathbb{N}} \mathcal{G}_n$ witnesses groupability of $\mathcal{G}$. Let for each n, $A_n = U_n^c \in \mathcal{A}_n$ and $\mathcal{B}_n = \mathcal{G}_n^c$. Then the collection $\{\mathcal{B}_n : n \in \mathbb{N}\}$ witnesses that $\{A_n : n \in \mathbb{N}\}$ a groupable element of Σ^+_E. Indeed, let $(S^c)^+$ be a Σ^+-neighborhood of E. Then $E \subset S^c$ implies $S \subset E^c$ so that there is k such that for all $n > k$ there exists a member $G_n \in \mathcal{G}_n$ with $S \subset G_n$, and consequently $G_n^c \in (S^c)^+$. This shows that $(S^c)^+ \cap \mathcal{B}_n \neq \emptyset$ for all but finitely many n. $\square$

The condition (2) in this theorem can be called the (Δ^+, Σ^+)-*selectively Reznichenko property* (of 2^X).

Let us recall that a space X is said to have *countable strong fan tightness* [22] if for each sequence $(A_n : n \in \mathbb{N})$ of subsets of X and each $x \in \bigcap_{n\in\mathbb{N}} \overline{A_n}$ there is a sequence $(x_n : n \in \mathbb{N})$ such that for each n $x_n \in A_n$ and $x \in \overline{\{x_n : n \in \mathbb{N}\}}$, i.e. if for each $x \in X$ the selection hypothesis $\mathsf{S}_1(\Omega_x, \Omega_x)$ is satisfied.

It was shown in [6] that $(2^X, \mathsf{Z}^+)$ (resp. $(2^X, \mathsf{F}^+)$) has countable strong fan tightness if and only if each open set $Y \subset X$ satisfies $\mathsf{S}_1(\Omega, \Omega)$ (resp. $\mathsf{S}_1(\mathcal{K}, \mathcal{K})$).

According to [17] a space X satisfies $\mathsf{S}_1(\Omega, \Omega^{gp})$ if and only if all finite powers of X have the *Gerlits-Nagy property* [11] (equivalently, the Hurewicz property as well as the Rothberger property). Recall that X has the *Rothberger property* [21] if it satisfies $\mathsf{S}_1(\mathcal{O}, \mathcal{O})$, where $\mathcal{O}$ is the family of open covers of X. X has the *Hurewicz property* [13] if for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of open covers of X there is a sequence $(\mathcal{V}_n : n \in \mathbb{N})$ such that for each n $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and each $x \in X$ belongs to $\cup\mathcal{V}_n$ for all but finitely many n.

Letting in Theorem 2 $\Delta = \Sigma = \mathbb{F}(X)$ (resp. $\Delta = \Sigma = \mathbb{K}(X)$) we obtain the following two important corollaries.

Corollary 3. *For a space X the following statements are equivalent:*

1. *$(2^X, \mathsf{Z}^+)$ has the Reznichenko property and countable strong fan tightness;*
2. *If Y is an open subset of X, then all finite powers of Y have the Gerlits-Nagy property.*

Corollary 4. *For a space X the following are equivalent:*

1. *$(2^X, \mathsf{F}^+)$ has the Reznichenko property and countable strong fan tightness;*
2. *Each open set $Y \subset X$ satisfies $\mathsf{S}_1(\mathcal{K}, \mathcal{K}^{gp})$.*

Similarly to the proof of Theorem 2 we can prove

Theorem 5. *Let X be a space and let Δ and Σ be subsets of 2^X closed for finite unions and containing all singletons. Then the following statements are equivalent:*

1. *2^X satisfies $\mathsf{S}_{fin}(\Delta_A^+, (\Sigma_A^+)^{gp})$ for each $A \in 2^X$;*
2. *Each open set $Y \subset X$ satisfies $\mathsf{S}_{fin}(O\Delta, O\Sigma^{gp})$.*

A space X is said to have *countable fan tightness* [1], [2] if for each $x \in X$ it satisfies $\mathsf{S}_{fin}(\Omega_x, \Omega_x)$, i.e. if whenever $(A_n : n \in \mathbb{N})$ is a sequence of subsets of X and $x \in \bigcap_{n\in\mathbb{N}} \overline{A_n}$ there are finite sets $B_n \subset A_n$, $n \in \mathbb{N}$, such that $x \in \overline{\bigcup_{n\in\mathbb{N}} B_n}$.

In [6] it was proved that $(2^X, \mathsf{Z}^+)$ (resp. $(2^X, \mathsf{F}^+)$) has countable fan tightness if and only if each open set $Y \subset X$ satisfies $\mathsf{S}_{fin}(\Omega, \Omega)$ (resp. $\mathsf{S}_{fin}(\mathcal{K}, \mathcal{K})$. A result in [17] states that all finite powers of of a space X have the Hurewicz property if and only if X belongs to the class $\mathsf{S}_{fin}(\Omega, \Omega)$.

As the corollaries of Theorem 5 we have:

Corollary 6. *For a space X the following statements are equivalent:*

1. *$(2^X, \mathsf{Z}^+)$ has both the Reznichenko property and countable fan tightness;*
2. *If Y is an open subset of X, then each finite power of Y has the Hurewicz property.*

Corollary 7. *For a space X the following are equivalent:*

(1) $(2^X, \mathsf{F}^+)$ *has the Reznichenko property and countable fan tightness;*
(2) *Each open set* $Y \subset X$ *satisfies* $\mathsf{S}_{fin}(\mathcal{K}, \mathcal{K}^{gp})$.

3. The Pytkeev property

A space X has the *Pytkeev property* if for each $A \subset X$ and each $x \in \overline{A \setminus \{x\}}$ there is a countable collection $\{A_n : n \in \mathbb{N}\}$ of (countable) infinite subsets of A which is a π-network at x, i.e. each neighborhood of x contains some A_n. This property was introduced in [20] and then studied in [18] (where the name Pytkeev space was used) and [9]. Pytkeev's property in function spaces $\mathsf{C}_p(X)$ was studied in [23].

Every (sub)sequential space has the Pytkeev property [20, Lemma 2] and every Pytkeev space has the Reznichenko property [18, Corollary 1.2].

In this section we consider the Pytkeev property in hyperspaces with Δ^+-topologies. We begin by the following general result.

Theorem 8. *If* X *is a space and* Δ *and* Σ *subsets of* 2^X *containing all singletons and closed for finite unions, then the following are equivalent:*

(1) 2^X *has the* (Δ^+, Σ^+)*-Pytkeev property, i.e. for each* $\mathcal{A} \subset 2^X$ *and each* $S \in Cl_{\Delta^+}(\mathcal{A} \setminus \{S\})$ *there is a family* $\{\mathcal{B}_n : n \in \mathbb{N}\}$ *of countable infinite subsets of* $\mathcal{A}$ *which is a* π*-network at* S *with respect to the* Σ^+*-topology.*
(2) *For each open set* $Y \subset X$ *and each* Δ*-cover* $\mathcal{U}$ *of* Y *there is a sequence* $(\mathcal{U}_n : n \in \mathbb{N})$ *of infinite subsets of* $\mathcal{U}$ *such that* $\{\cap \mathcal{U}_n : n \in \mathbb{N}\}$ *is a (not necessarily open)* Σ*-cover of* Y.

Proof. (1) $\Rightarrow$ (2): Let $\mathcal{U}$ be a Δ-cover of Y. Then $\mathcal{A} := \{U^c : U \in \mathcal{U}\}$ is a subset of 2^X and $Y^c \in Cl_{\Delta^+}(\mathcal{A})$. Apply (1) to find a sequence $(\mathcal{B}_n : n \in \mathbb{N})$ of countable infinite subsets of $\mathcal{A}$ such that the set $\{\mathcal{B}_n : n \in \mathbb{N}\}$ ia a Σ^+-π-network at Y^c. For each n denote by $\mathcal{U}_n$ the subset $\{U : U^c \in \mathcal{B}_n\}$ of $\mathcal{U}$. We claim that $\{\cap \mathcal{U}_n : n \in \mathbb{N}\}$ is a Σ-cover of Y. For any $S \subset Y$ with $S \in \Sigma$, the set $(S^c)^+$ is a Σ^+-neighborhood of Y^c and therefore there is $m \in \mathbb{N}$ such that $\mathcal{B}_m \subset (S^c)^+$. Therefore, for each $U^c \in \mathcal{B}_m$ we have $U^c \subset S^c$, i.e. $S \subset \cap\{U : U^c \in \mathcal{B}_m\}$. This means that the collection $\{\cap \mathcal{U}_n : n \in \mathbb{N}\}$ is indeed a Σ-cover of Y.

(2) $\Rightarrow$ (1): Let $\mathcal{A}$ be a subset of 2^X and let $S \in Cl_{\Delta^+}(\mathcal{A} \setminus \{S\})$. Then S^c is an open subset of X and the family $\mathcal{U} \equiv \mathcal{A}^c := \{A^c : A \in \mathcal{A}\}$ is an open Δ-cover of S^c. Apply (2) to the set S^c and its Δ-cover $\mathcal{U}$ and choose infinite sets $\mathcal{U}_n \subset \mathcal{U}$, $n \in \mathbb{N}$, such that the family $\{\cap \mathcal{U}_n : n \in \mathbb{N}\}$ is a Σ-cover of S^c. Put for each n

$$\mathcal{A}_n = \{U^c : U \in \mathcal{U}_n\}.$$

Then each $\mathcal{A}_n$ is an infinite subset of $\mathcal{A}$. We prove that the collection $\{\mathcal{A}_n : n \in \mathbb{N}\}$ is a π-network at $S \in (2^X, \Sigma^+)$.

Let $(E^c)^+$, $E \subset X$ and $E \in \Sigma$, be a Σ^+-neighborhood of S. Then $S \subset E^c$ implies $E \subset S^c$ so that there is some $i \in \mathbb{N}$ with $E \subset \cap \mathcal{U}_i$. Further we have $\mathcal{A}_i \subset (E^c)^+$, i.e. (1) holds. □

As consequences of this theorem we obtain the following two results.

Corollary 9. *For a space X the following statements are equivalent:*

(1) $(2^X, \mathsf{Z}^+)$ *has the Pytkeev property;*
(2) *For each open set $Y \subset X$ and each ω-cover $\mathcal{U}$ of Y there is a sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of infinite countable subsets of $\mathcal{U}$ such that $\{\cap \mathcal{U}_n : n \in \mathbb{N}\}$ is an (not necessarily open) ω-cover of Y.*

Corollary 10. *For a space X the following statements are equivalent:*

(1) $(2^X, \mathsf{F}^+)$ *has the Pytkeev property;*
(2) *For each open set $Y \subset X$ and each k-cover $\mathcal{U}$ of Y there is a sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of countable infinite subsets of $\mathcal{U}$ such that $\{\cap \mathcal{U}_n : n \in \mathbb{N}\}$ is a (not necessarily open) k-cover of Y.*

Problem 11. *If $(2^X, \mathsf{F}^+)$ has the Pytkeev property, is $(2^X, \mathsf{F}^+)$ sequential? What about $(2^X, \mathsf{Z}^+)$?*

Remark A It is known that the space $(2^X, \mathsf{F})$ is compact with no assumptions on X and that a Hausdorff space X is locally compact if and only if $(2^X, \mathsf{F})$ is Hausdorff. According to a result from [18], each compact Hausdorff space of countable tightness is a Pytkeev space. On the other hand, in [12] (see also [4]) it was shown that for a locally compact Hausdorff space X the tightness of $(2^X, \mathsf{F})$ is countable if and only if X is hereditarily separable and hereditarily Lindelöf. So we have:

Theorem 12. *For a locally compact Hausdorff space X the following assertions are equivalent:*

(a) $(2^X, \mathsf{F})$ *has countable tightness;*
(b) $(2^X, \mathsf{F})$ *has the Reznichenko property;*
(c) $(2^X, \mathsf{F})$ *has the Pytkeev property;*
(d) *X is both hereditarily separable and hereditarily Lindelöf.*

Remark B According to results from [3] and [7] one can suppose that in some models of ZFC (in which each compact Hausdorff space of countable tightness is sequential) we have: For a locally compact Hausdorff space X each of the conditions (a)–(d) in the previous theorem is equivalent to the assertion $(2^X, \mathsf{F})$ is a sequential space.

Remark C Let us consider a selective version of the Pytkeev property. Call a space X *selectively Pytkeev* if for each sequence $(A_n : n \in \mathbb{N})$ of subsets of X and each point $x \in \bigcap_{n\in\mathbb{N}} \overline{A_n \setminus \{x\}}$ there is an infinite family $\{B_n : n \in \mathbb{N}\}$ of countable infinite sets which is a π-network at x and such that for each n $B_n \subset A_n$. Then one can prove the statements similar to those in Theorem 8 and Corollaries 9 and 10.

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Department of Mathematics, Faculty of Sciences and Mathematics, University of Niš, Višegradska 33, 18000 Niš, Serbia

E-mail address: `lkocinac@ptt.yu`